Brief Announcement: An Improved Distributed Approximate Single-Source Shortest Paths Algorithm

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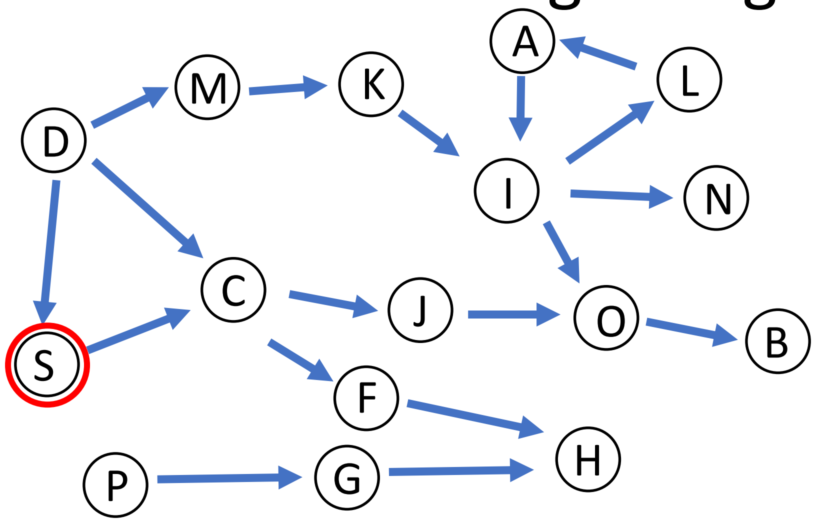
The CONGEST model

- Processors with unique IDs are modeled as n nodes.
- Synchronous rounds (global clock)
- In each round, every node sends (at most) one message with size O(log n) to each neighbor.
- At the end of the algorithm, every node has to compute some information.
- Complexity Measure: number of rounds

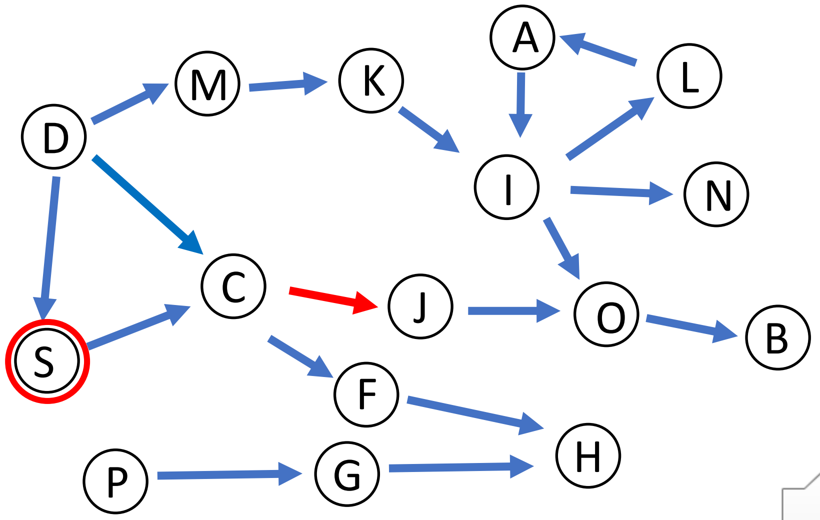
The approximate single source shortest path problem

- Given graph G and a source node s, compute the $(1+\epsilon)$-approximate shortest path distance from s to all other nodes.
  - Problem: Compute $\widehat{dist_G}(s,v)$ such that $dist_G(s,v) \le \widehat{dist_G}(s,v) \le (1+\epsilon) \cdot dist_G(s,v)$, where $dist_G(s,v)$ is the shortest path distance from s to v.
- The graph G is directed and the edge weight is non-negative real number.



The approximate single source shortest path(ASSSP) problem in the CONGEST model

- The nodes of the graph G are processors in the CONGEST model.
- The graph G is directed, but messages can be sent in both directions.
- D is the diameter of the undirected graph. Sending one message from one node to another takes O(D) rounds.



Related work

Algorithms	rounds	Remark
Peleg and Rubinovic 2000	$\tilde{\Omega}\left(\sqrt{n}+D\right)$	Lower bound for directed and undirected approximate SSSP
Nanongkai 2014 Forster and Nanongkai 2016	$\tilde{O}\left(\left(\sqrt{n}D^{\frac{1}{4}}+D\right) lg\, W\ /\epsilon\right)$	directed approximate SSSP
Chechik and Mukhtar 2020	$\tilde{O}\left(\sqrt{n}D^{\frac{1}{4}}\, lg^2\, W\right)$	directed exact SSSP(integer weight)
This paper	$\tilde{O}\left(\left(\sqrt{n}+D+n^{\frac{2}{5}+o(1)}D^{\frac{2}{5}}\right) lg\, W\ /\epsilon^2\right)$	directed approximate SSSP

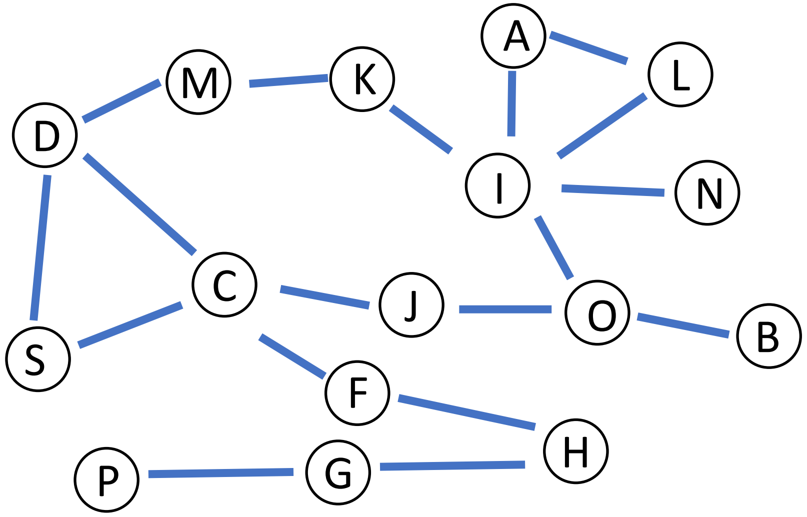
n = number of nodes, m = number of edges, D = diameter, W =heaviest edge weight/lightest edge weight

Related work --- some parameter

- If W is polynomial bounded and ϵ is constant, then previous best result is $\tilde{O}\ (\sqrt{n}D^{\frac{1}{4}} + D)$, and our result is $\tilde{O}\ (\sqrt{n} + D + n^{\frac{2}{5}+o(1)}D^{\frac{2}{5}})$.
- $D = o(n^{1/4})$, then $\tilde{O}\left(\sqrt{n}D^{\frac{1}{4}}\right) \rightarrow \tilde{O}(\sqrt{n})$
- $D = O\left(n^{\frac{2}{3}}\right)$, for example, $D = \theta(\sqrt{n})$, $\tilde{O}(n^{0.625}) \rightarrow \tilde{O}\ (n^{0.6+o(1)})$.

The skeleton graph approximate SSSP

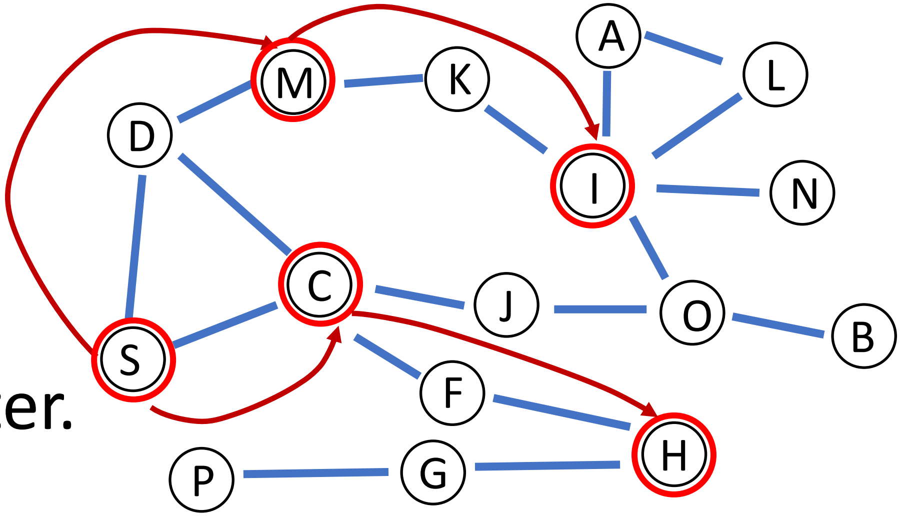
- Given skeleton graph G_s, which contains $\tilde{O}(\alpha)$ nodes, each node of G_s knows its incoming and outgoing edges.
- G_s is a virtual directed graph over some nodes.



$\tilde{O}(\alpha)$ nodes.

The skeleton graph approximate SSSP

- Given skeleton graph G_s, which contains. $\tilde{O}(\alpha)$ nodes, each node of G_s knows its incoming and outgoing edges.
- G_s is a virtual directed graph over some nodes. How we construct G_s doesn't matter.
- Solving ASSSP over skeleton graph ⇒ Solving ASSSP over whole graph[Forster and Nanongkai 2016]



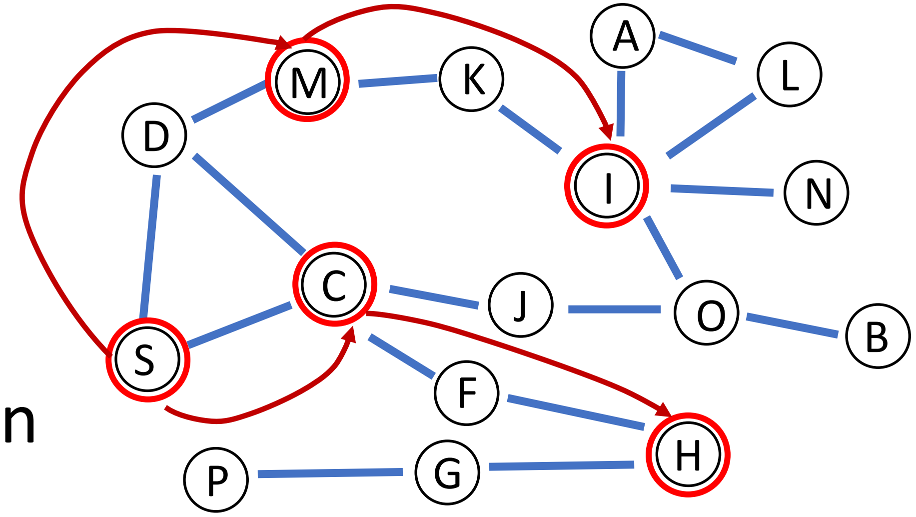
$\tilde{O}(\alpha)$ nodes.

Improve the skeleton graph approximate SSSP

- Solving approximate SSSP over skeleton graph[Forster and Nanongkai 2016]

$$\tilde{O}\left(\alpha\rho + \frac{D\alpha}{\rho}\right) \rightarrow \tilde{O}\left(\alpha\rho^2 + \frac{D\sqrt{\alpha}}{\rho}\right)$$

- Difficulty: no communication link between any edge of the skeleton graph.

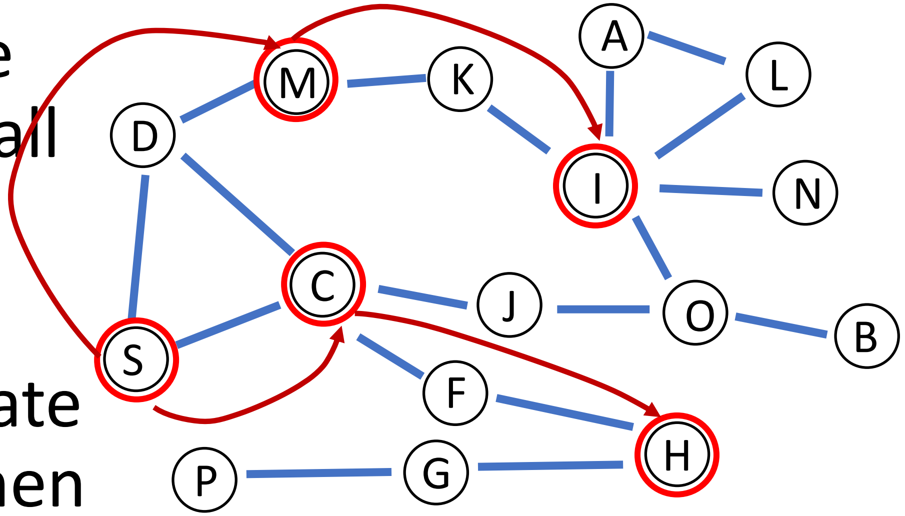


$\tilde{O}(\alpha)$ nodes.

ρ is a parameter

Key observation for the skeleton graph ASSSP

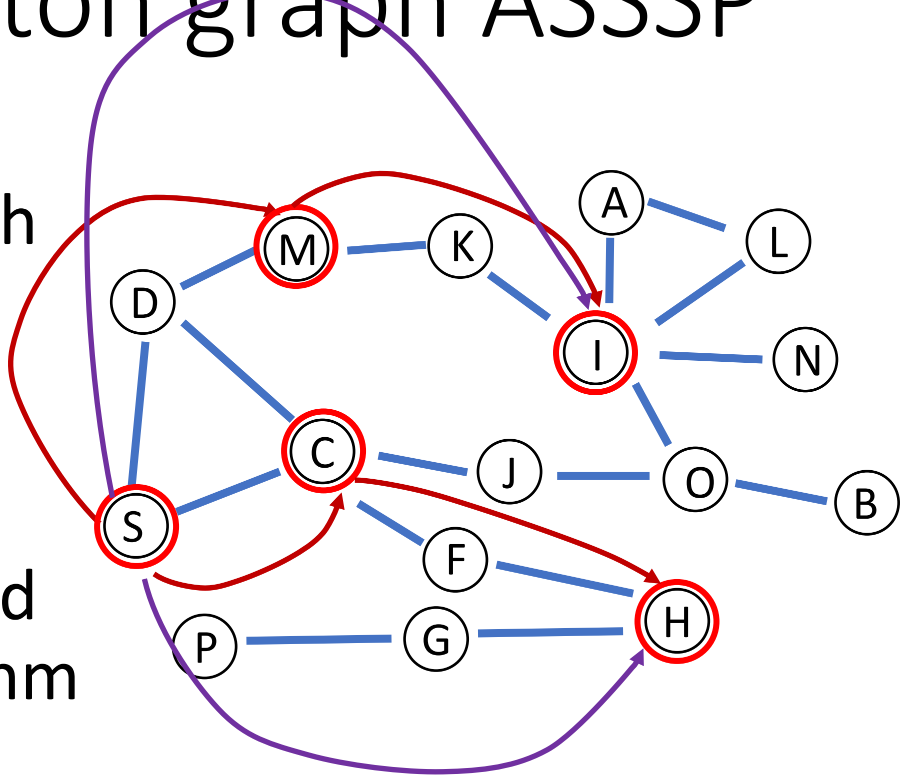
- Since we only care about the approximate SSSP, we could round up each edge to small integer value and thus the edge weight could be treated as 1. [KS97]
- If from s to all other nodes, the approximate shortest path contains at most k edges, then simulating BFS is enough. Each level takes $O(D)$ rounds and in total takes $O(Dk + \alpha)$.



Skeleton graph

Algorithm Idea for the skeleton graph ASSSP

- The idea is to add more edges to the graph until k is small.
- CFR2020 could do in parallel model, we adapt it to distributed model.
- Jambulapati, Liu, and Sidford 2019 showed a similar transformation, but their algorithm is for reachability problem and our algorithm gets better running time.



New skeleton graph: k becomes smaller.

Conclusion

- Combine the framework of Forster, Nanongkai 2016 and parallel shortest path algorithm CFR2020
- An algorithm for distributed approximate single shortest paths problem